Real Time Vehicle State Estimation Using Low Cost Inertial Measurement Unit

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Abstract
According to statistics, accidents involving heavy vehicles are generally more serious than those of light vehicles. For the proper functioning of vehicle stability control under dynamic road conditions and driver's inputs, precise estimations of vehicle states are necessarily required. In particular, information on the roll angle is critical to vehicle handling and safety control. Commercially available sensors which measure the roll angle are not cost effective that is the reason estimation methods that use available sensor measurements and vehicle dynamics models are necessarily required. This paper proposes a novel methodology to estimate the roll angle of vehicle. A roll angle observer is designed using the bicycle model and the kinematic model. In the first step of research we used Kalman filter to extract roll angle and gravity compensated lateral acceleration from kinematic model. After which, we used these estimations as input for our dynamic model to estimate roll dynamics. Kalman filter and Sliding mode observer is implemented using simplest roll dynamics model to measure the roll angle of a vehicle and the validation of results is carried using commercial software, CarSim®.(CarSim, Michigan US) under variety of maneuvers.

Keywords: State observer; vehicle dynamics; state estimation; roll angle

1. Introduction

In current era, the primary objective of road transport systems is to reduce the number of accidents and for the same reason, vehicles in the market are equipped with sophisticated control systems, such as ESC (Electronic Stability Control) and RSC (Roll Stability Control) [1,2] for the improvement of vehicle safety standards. The discussed systems rely on complete knowledge of vehicle behavior in advance during different conditions and maneuvers for the proper actuation of the control system [3–5]. In particular the knowledge about the roll angle of the vehicle is of significant importance in RSC systems. Rollover accidents are responsible for nearly 33% of all deaths from passenger vehicle crashes [6].

The success of RSC system depends on the vehicle roll angle information. Dual-antenna GPS can be used to directly measure vehicle roll angle but the equipment cost of such system is high [7]. For this reason, roll angle of a vehicle needs to be estimated using available measurements and a cost-effective system [8,9]. There are three approaches found in literature currently [6,10] which can be used to estimate attitude of a vehicle: (1) indirect approach; (2) vehicle dynamics model approach; (3) additional sensor aided approach. The indirect approach uses vehicle sensors such as wheel speed sensors and inertial measurement unit (IMU) [9]. The in-vehicle sensor approach is currently the cheapest solution for vehicle attitude estimation problem but it suffers from cumulative integration errors due to accelerometer bias and gyro drift. The vehicle modeling approach [11] requires the accurate vehicle dynamics model as well as vehicle parameters in addition to bias compensation for precise estimation. The additional sensor-aided approach [12] such as using a vision sensor can provide the accurate heading angle of the vehicle. However, the update rate of the vision sensor is too slow, and the failure of camera can frequently occur due to the influence of road conditions and extreme weather changes.

In [13], an algorithm vehicle roll angle estimation algorithm is proposed which utilizes the measurements obtained from suspension deflection sensors and accelerometers. However, the estimation method lacks precision. Furthermore, suspension deflection sensors in market are often quite expensive and are typically not designed for majority of standard vehicles. In [10], a dynamic observer is proposed which used the information obtained from a lateral accelerometer and a gyroscope. However, the error produced in the transient response of estimated vehicle roll angle is high, neither model nor measurement noise is taken into account in the mentioned algorithm. An online accelerometer bias estimation algorithm is proposed in [14]. An estimation method based on lateral velocity and attitude has been proposed for automated driving vehicle in [15]. The stated method fused the information obtained from six DOF IMU and vehicle dynamics, and it can run autonomously without aid from extra information. The root mean square error (RMSE) for roll angle obtained through this technique remained at 0.6 degree. There are many non-linear attitude estimation techniques discussed by [16] such as QUEST, recursive QUEST, Extended Kalman Filter, multiplicative Kalman Filter, backward smoothing extended Kalman filter. Discrepancies of each individual algorithm were discussed based on computational burden, convergence time and noise response. Filters such as extended Kalman rely on priori estimate and may not produce accurate results if priori estimate is not accurate or strongly nonlinear dynamics intervene. Similarly, unscented filters might be attractive for systems with nonlinear dynamics but lag in terms of computational burden. Dimensionality constraint is common among particle filters if more than few parameters need to be estimated. The discussed systems show promising results in complex systems when higher computational power and accurate priori estimates are available Therefore, simple model based observers along with attitude estimation algorithms are proposed in this research to reduce computational power and convergence time.

Global Positioning System (GPS) is not feasible due to signal outage in urban regions, tunnels and Parking lots etc. [17, 18]. Two primary requirements need to be met for the sensor design for driving applications. First, the sensor should be capable enough to provide continuous measurement under all circumstances. Secondly, sensors must show robust performance under all vehicle maneuvers regardless of severity and duration of maneuver. Low cost inertial navigation systems (INS) tend to produce large estimation errors and signal severity and time increases [19]. Automotive Gyroscopes are the simplest and most precise solution for vehicle attitude estimation. Attitude angle can be obtained by integrating angular velocities using tri-axial gyroscope. Integration process itself is used to accumulate any noise or bias in sensor measurements that can drift with time. Fiber-optic and ring laser technology is used in automotive gyroscopes to deliver the accuracy necessary for driving applications. However, they are quiet expensive and sometimes their cost is comparable to that of the vehicle itself.

In this paper, we addressed the main issues related to state estimation of vehicle roll angle. Attitude estimation in case of ground vehicles is challenging due to terrain roughness, road disturbances and noise. Model based observers are often too complex to implement and require powerful onboard processor to deal with computation cost. Vehicle dynamics simulation is narrowed down using simple models which are computationally less intensive as compared to complex models.

The proposed methods are easy to implement and fast since they are free from iteration. In order to eliminate the significant effects of gyro bias, we have suggested a bias compensation which is merged with filtering architecture. Experimental validation is carried out on hardware in real time environment to show the performance comparison Accuracy of results show that the proposed filters are competitive in performance to Kalman filter and higher order observers. Suggested algorithms promise high accuracy and low computational cost in less convergence time.

Size and cost of inertial sensors shrunk since the introduction of MEMS [20]. Miniature gyroscopes and accelerometers are available at a very low cost these days but their performance is too much degraded when used in automotive applications. Automotive grade gyroscope typically produces 1°/h drift whereas; a MEMS gyroscope performance deteriorates at 70°/h [21]. Moreover, temperature variations play vital role in this bias as well [22]. Currently, much work is been done in order to improve the performance and quality of MEMS gyroscope to make them more robust to vibrations and noise sources for automotive applications [23].

2. Materials and Methods

The InvenSense MPU-6050 (TDK InvenSense, US) is a sensor with low cost that contains a MEMS gyroscope and a MEMS accelerometer on a single chip. Standard I2C-bus Interface is used for communication. In this research additional processing hardware is attached to MPU-6050 and interfacing is done with the MATLAB® using instrumentation and control toolbox. This sensor is mounted at approximate location of center of gravity of vehicle. The vehicle is driven in different urban conditions to acquire results.

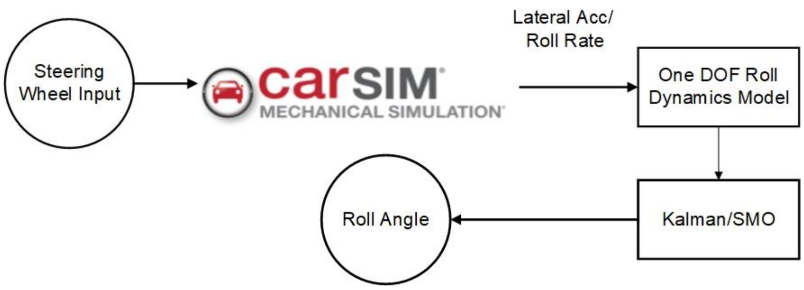

Figure 1. Dynamics based estimation architecture

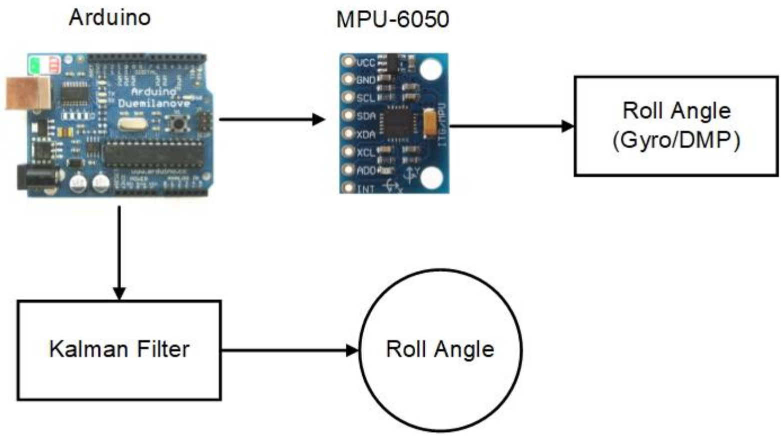

Figure 2. Kinematics Based Estimation Architecture

2.1. Mathematical Modeling and Validation

2.1.1 Mathematical Modeling

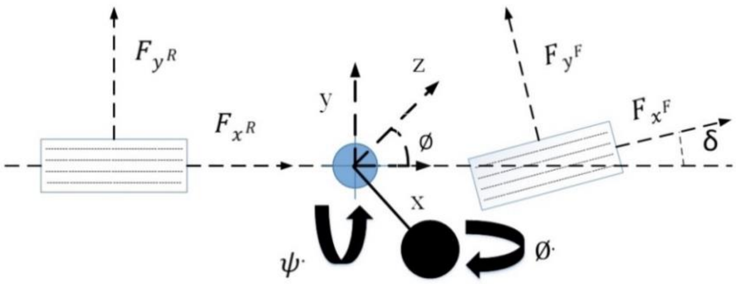

Figure 3. Single Track Vehicle Model

WE have used bicycle model for implementation of our algorithm (see Figure 3). Considering roll dynamics we can write equation for lateral motion as,

$$\mathbf{m a_y = F y_f + F y_r}, \tag{1}$$

In eq(1) m is the mass of vehicle. Lateral acceleration is denoted by $\mathbf{a_y}$, $\mathbf{Fy_f}$ denotes the front tire force and $\mathbf{Fy_r}$ is rear tire force. Yaw rate can be expressed as,

$$\mathbf{I_z \ddot{\psi} = l_f F y_f - l_r F y_r}, \tag{2}$$

$\mathbf{I_z}$ is the yaw inertia in eq(2), $\ddot{\boldsymbol{\psi}}$ is the yaw rate, $\mathbf{l_f}$ is distance of front from center of gravity and $\mathbf{l_r}$ is the distance of rear from center of gravity of vehicle. Roll rate can be written as,

$$\mathbf{I_{xx} \dot{\phi} + C_q \dot{\phi} + K_q \phi = m h a_y} \tag{3}$$

According to eq(3) Roll moment of inertia is $\mathbf{I_{xx}}$, $\dot{\boldsymbol{\phi}}$ denotes the roll rate, and $\boldsymbol{\phi}$ is the roll angle of vehicle, $\mathbf{m}$ represents mass of vehicle, h is the height from center of gravity. $\mathbf{C_q}$ is expressed as compliance and $\mathbf{K_q}$ denotes the stiffness coefficient. Now we will substitute the value of $\mathbf{a_y}$,$\mathbf{Fy_f}$ and $\mathbf{Fy_r}$, the expression for lateral acceleration becomes,

$$\dot{\mathbf{v}}_{\mathbf{y}} = \frac{\mathbf{C_f}}{\mathbf{m}\left[\boldsymbol{\delta} - \frac{\mathbf{v_y} + \mathbf{Lf}(\dot{\boldsymbol{\psi}})}{\mathbf{v_x}}\right]} + \frac{\mathbf{C_r}}{\mathbf{m}\left[- \frac{\mathbf{v_y} - \mathbf{Lr}(\dot{\boldsymbol{\psi}})}{\mathbf{v_x}}\right]} \tag{4}$$

$\dot{\mathbf{v}}_{\mathbf{y}}$ is the lateral acceleration in eq(4), $\boldsymbol{\delta}$ is the steer angle $\dot{\boldsymbol{\psi}}$ is the yaw rate, $\mathbf{v_x}$ is the longitudinal velocity and $\mathbf{v_y}$ is the lateral velocity. Yaw acceleration can be represented in eq(5) as,

$$\ddot{\boldsymbol{\psi}} = \frac{\mathbf{aC_f}}{\mathbf{I_{zz}}\left[\boldsymbol{\delta} - \frac{\mathbf{v_y} + \mathbf{Lf}(\dot{\boldsymbol{\psi}})}{\mathbf{v_x}}\right]} - \frac{\mathbf{bC_r}}{\mathbf{I_{zz}}\left[- \frac{\mathbf{v_y} - \mathbf{Lr}(\dot{\boldsymbol{\psi}})}{\mathbf{v_x}}\right]} \tag{5}$$

$\ddot{\boldsymbol{\psi}}$ is the yaw acceleration, $\mathbf{I_{zz}}$ represents the yaw moment of inertia, $\mathbf{v_x}$ is the longitudinal velocity and $\mathbf{v_y}$ is the lateral velocity. Considering the single track model roll acceleration can be expressed as,

$$\ddot{\boldsymbol{\phi}} = \frac{\mathbf{mh}}{\mathbf{I_{xx}}[\mathbf{v_{y^{\cdot}}} + \mathbf{v_x}\boldsymbol{\psi}^{\cdot}]} - \frac{\mathbf{C_q}\boldsymbol{\phi}^{\cdot}}{\mathbf{I_{xx}}} - \frac{\mathbf{K_q}\boldsymbol{\phi}}{\mathbf{I_{xx}}} \tag{6}$$

Where $\ddot{\boldsymbol{\phi}}$ is the roll acceleration, $\mathbf{I_{xx}}$ is the roll moment of inertia

Online Parameter Estimation:

State observer can be best described as a system that can be used to provide estimate of the internal states of a real time system. It utilizes available input measurement of system to provide estimate of output under consideration which cannot be directly measured.

Kalman Filter

A Kalman filter [24] is a stochastic system modeling technique that can be applied to a controls or data processing problem when deterministic models and techniques are not sufficient.

System to be estimated

The Kalman filter works out the state estimate of a discrete time system, which can be denoted by a linear stochastic difference equation and its measurement equation as under:

$$\boldsymbol{x_{k+1}} = \boldsymbol{A_k x_k} + \boldsymbol{Bu_k} + \boldsymbol{w_k} \quad (7)$$

$$z_k = H_k x_k + v_k \quad (7)$$

Kalman Filter can update in two steps.

1. Time update or predictor equations for increasing time.

$$\hat{x}_{k+1}^{-} = A_k \hat{x}_k + Bu_k \tag{8}$$

$$P_{k+1}^{-} = A_k P_k A_k^T + Q_k \tag{9}$$

2. Measurement update or corrector equations for adjusting new measurement in *a priori* estimate.

$$K_k = (P_k^- H_k^T + R_k)^{-1} \tag{10}$$

$$\hat{x}_k = \hat{x}_k^- + K(z_k - H_k \hat{x}_k^-) \tag{11}$$

$$P_k = (I - K_k H_k) P_k^- \tag{12}$$

After each update, the above two steps are repeated.

Sliding Mode Observer:

Sliding mode observers have unique properties, in that the ability to generate a sliding motion on the error between the measured plant output and the output of the observer ensures that a sliding mode observer produces a set of state estimates that are precisely commensurate with the actual output of the plant.

The state space can be written as,

$$\dot{x} = Ax + Bu \tag{13}$$

$$y = Cx \tag{14}$$

$$T_c = \begin{bmatrix} N_c^T \\ C \end{bmatrix} \tag{15}$$

where the columns of $N_c \in \mathbb{R}^{n\times(n-p)}$ span the null space of C.

The non-singular transformation, $CT_c^{-1} = [0 \quad I_p]$ becomes the new distribution matrix. The other system matrices can be expressed as, $T_c A T_c^{-1} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$ and $T_c B = \begin{bmatrix} B_2 \\ B_2 \end{bmatrix}$ where $T_c x = \begin{bmatrix} x \\ y \end{bmatrix}$.

Then, the nominal system can be written as under.

$$\dot{x} = A_{11}x + A_{12}y + B_1 u \tag{16}$$

$$\dot{y} = A_{21}x + A_{22}y + B_2 u \tag{17}$$

Considering the state matrix represented in Equation (16),

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{K_q}{I_{xx}} & -\frac{C_q}{I_{xx}} \end{bmatrix}$$

Similarly,

$$\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{mh}{I_{xx}} \end{bmatrix}$$

The proposed Utkin observer has the form,

$$\dot{\hat{x}} = A_{11}\hat{x} + A_{12}\hat{y} + B_1 u + Lv \tag{18}$$

$$\dot{\hat{y}} = A_{21}\hat{x} + A_{22}\hat{y} + B_2 u - v \tag{19}$$

Vector L is computed in such a way that $A_{11} + LA_{21}$ lies in spectrum C. The discontinuous vector v is defined by,

$$v = M\,\mathrm{sgn}(\hat{y} - y) \tag{20}$$

The value of variable M used in experiments remained 0.1.

Results Comparison with CARSIM

We have used CarSim® software to simulate vehicle response under different road maneuvers. Vehicle handling response depend on steer profile and velocity of vehicle. CarSim® database contains large number of vehicles for simulation purpose. The user chooses a vehicle of his/her choice and simulates the dynamic response of the vehicle by giving different inputs to the vehicle.

A step steer input of 70 deg is fed to the Roll dynamics model and the results for Roll angle is shown below in Figure 4. These results show that there is error of 6% between model and CarSim results which is within the acceptable limits to capture the major dynamics of the system.

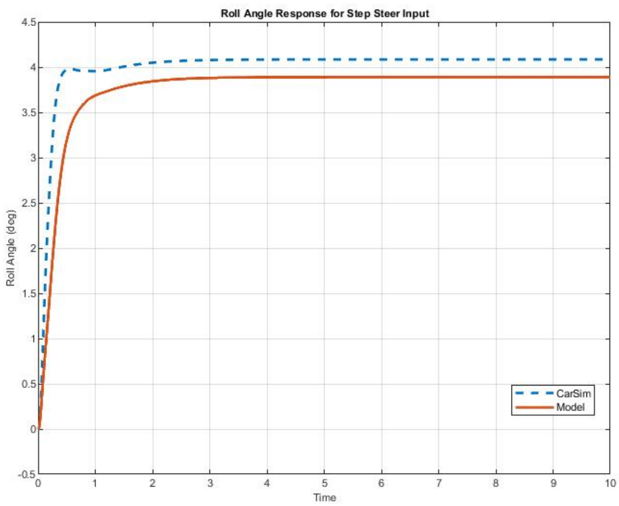

Figure 4: Step Steer response for Roll Angle

The model is fed with the ISO double lane change maneuver and the results for the Roll angle is seen in Figure 5.

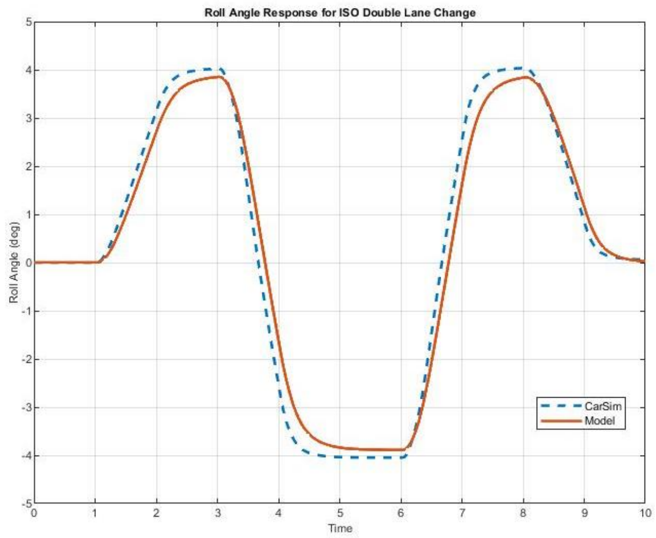

Figure 5: Roll Angle Response for ISO Double Lane change Maneuver

Comparison between Kalman and Sliding Mode Observer

A step steer input of 70 deg is given to both observers and CarSim as seen in Figure 6. It has been observed that the sliding mode observer kept track of roll angle better than Kalman filter during step steer input and root mean square error for sliding mode observer is 0.0807 degrees whereas Kalman filter produced root mean square error of 0.19 degrees.

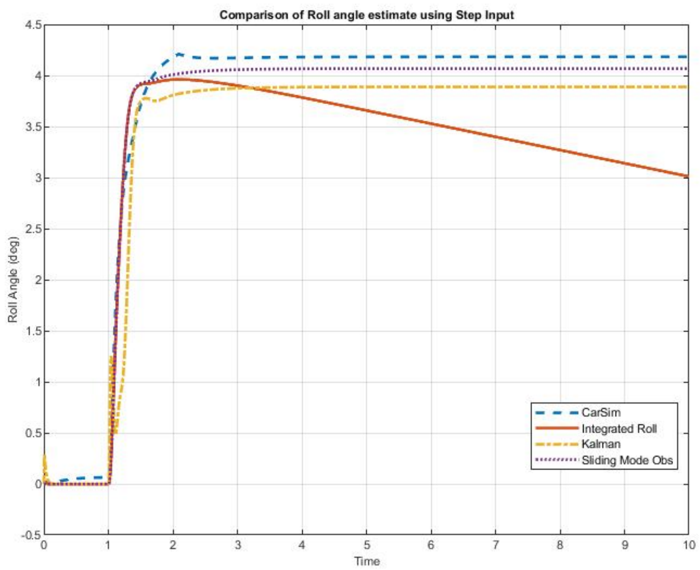

Figure 6: Observer Results for step Input estimate for Roll Angle

Similarly, in case of ISO Fish hook maneuver both the observers are fed with roll rate and lateral acceleration as input. It has been observed that the sliding mode observer kept track of roll angle better and root mean square error between sliding mode observer and CarSim is found to be 0.0873 degrees and 0.1459 degrees respectively.

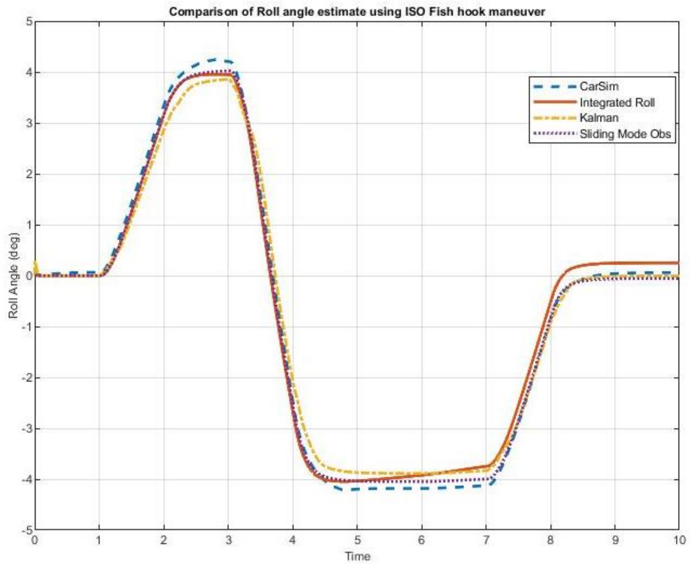

Figure 7: Observer Results for ISO Fish hook Input estimate for Roll Angle

Finally to check the performance of observer further it is fed with sine input. The Kalman filter shows slight deviation while tracking the roll angle but sliding mode observer tracked the roll angle better and the root mean square for sliding mode observer turned out to be 0.1538 and Kalman filter produced root mean square error of 0.4472.

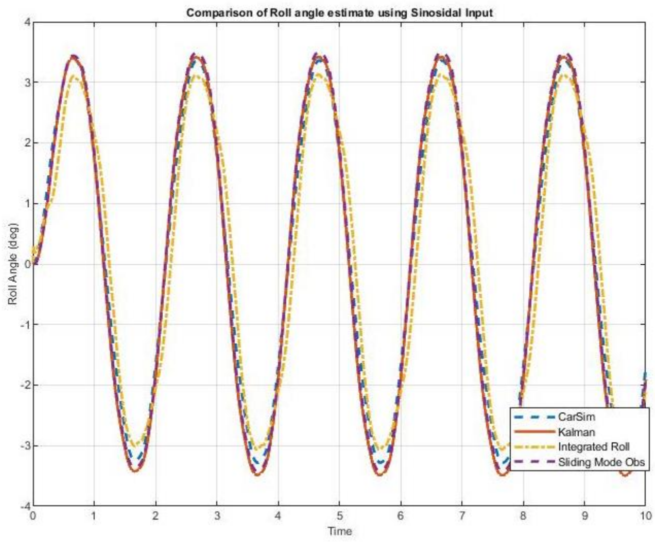

Figure 8: Observer Results for sine input estimate of Roll Angle

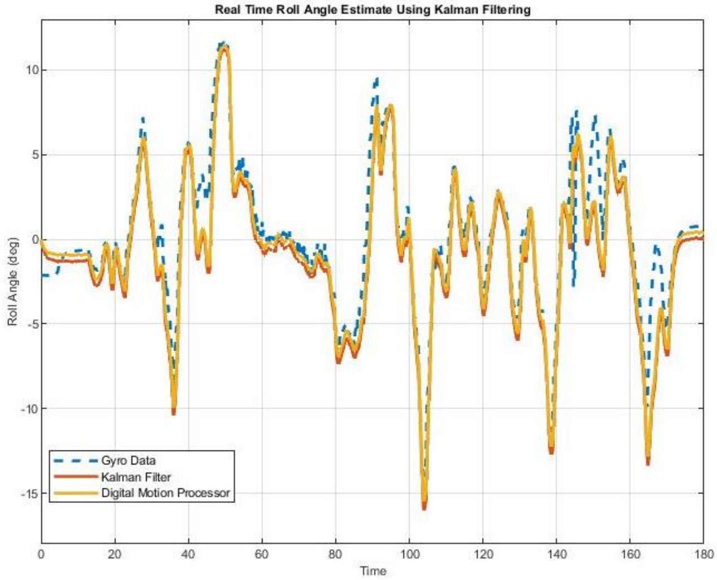

Figure 9: Real Time Roll angle estimate using Kalman Filtering

4. Discussion

Roll angle estimation involving a ground vehicle is challenging due to rough terrain and noise. A linear Kalman filter for roll angle estimation of ground vehicle is proposed. This method separates the correction of attitude from heading. Previous researches utilize the data from sensor dynamics, but the tests have not been performed in real time dynamic environment such as bumpy road where noise removal plays critical role in estimations. Our proposed filter is fast, since it is free of iteration. To decrease significant effects of bias imposing on gyroscope, a bias compensation is merged with filtering architecture. Experiment for efficacy of algorithm is carried out in real time environment to show the performance comparison with in built digital motion processor of sensor. Results show that the proposed filter can reach the accuracy of higher order observers. Successfully tested in ordinary road conditions we find a balance between estimation accuracy and time consumption. Compared with iterative methods, the proposed filter has much less convergence time.

Finally, the implementation of proposed algorithm is carried out on low cost hardware that can be easily mounted on standard commercial vehicle with addition of marginal cost and based on attitude information Anti Roll Systems can be designed and implemented.

5. Conclusions

Real Time implementation in this research is carried out using Multi axis gyroscope and accelerometer (MPU-6050). The estimation of Roll angle using this approach is used to remove the gravity component from lateral acceleration prior to estimation of vehicle's lateral velocity. Proposed estimation methods are simple and cost effective as they are not dependent on tire models or complex vehicle models. The major advantage of the techniques investigated is fast and accurate estimation. The RMSE remained less than 0.7 degree. The algorithm is tested in ordinary driving conditions with speeds varying from 0 to 70 Km/hr. The estimate of roll angle along with lateral acceleration in real time environment can aid in different tire road phenomenon investigation. This simple technique can be coupled with sliding mode observer to provide highly accurate estimate of vehicle dynamics. Finally, it can be safely claimed that in presence of ordinary road conditions this technique can easily provide accurate estimate of roll angle by adding marginal cost to the user.

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Conflicts of Interest: The authors declare that the research was conducted in the absence of any commercial or financial relationship that could be construed as a potential conflict of interest.

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