Stress Analysis of Flat and Conical Roof Cylindrical Tank Due to Seismic Sloshing

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Abstract
The objective of this article is to perform the seismic analysis of above ground cylindrical steel storage tank. This study has been performed numerically using ANSYS Workbench. Fluid Structure Interaction (FSI) analysis is done for the flat roof and conical roof of a cylindrical tank. A steel storage Tank of 90 m^3 capacity is used which is subjected to sinusoidal base excitation. In response to base excitation, the sloshing phenomena have been studied e.g., maximum sloshing wave height, hydrodynamic pressure variations and as a reaction the stresses produced in the tank. When the flat roof cylindrical steel tank is subjected to convective resonance it results in roof damage as equivalent stress value crosses the yield strength of tank material. Whereas when the flat roof is modified to the conical, the maximum stress decreased about 2.6 times. Conical roof design proves to be seismically safe even under large amplitude resonant seismic excitations.

Keywords
Cylindrical Tank, Seismic Sloshing, FSI Analysis, Convective Resonance, Roof Damages

Introduction

Liquid storage tanks are strategically significant thin-walled structures. They are used in various industries for storing water, dangerous chemicals, and petroleum etc. They should be seismically safe, contrary to other structures the seismic analysis of these structures requires special considerations because of fluid-structure interaction which makes them complicated during an earthquake event. Therefore, their structural behaviour should be well understood to protect them from the devastating effects of seismic loading. During the earthquake, the liquid inside the tank experiences horizontal acceleration and exerts hydrodynamic forces to the tank walls.

To determine the magnitude of these hydrodynamic forces Housner [1] simplified the problem by proposing an equivalent mechanical model by representing the tank fluid system as an equivalent spring-mass model. According to this model, the total mass of water is divided into two parts, one component is situated in the lower region of the tank which moves in unison with the tank walls called impulsive water mass and another component moves relative to the tank wall and exhibits sloshing called the convective mass [2]. Correspondingly the pressure imposed by impulsive mass is called impulsive pressure and convective mass will induce convective pressure to tank walls [3].

The equivalent mechanical models are widely used by different design codes i.e., IITK-GSDMA guidelines for seismic design of liquid storage tanks [4], API-650 [5], and Eurocode-8 [6] etc. These standards help to estimate the damage situations such as uplifting, base shear, overturning moment, and hydrodynamic hoop stresses [7]. The cylindrical steel tanks are thin-walled engineering structures made up of thin-perimeter walls therefore mostly they perform poorly under earthquake loading and the response of tank structure became non-linear under strong seismic excitation hence structural damage may not be assessed easily. The main seismic damages are buckling of the slender tank, shell rupture, and rood damage etc. [8]. Veletsos add the effects of shell flexibility on the hydrodynamic pressure and fluid

structure interaction [9]. Veletsos approach is used in Eurocode-8 to estimate the structural response [6]. The total sloshing pressure have been evaluated using expression given in Eurocode-8 and it uses the absolute sum approach to combine impulsive and convective pressure terms to get the combined sloshing effect [10].
This paper focuses on the stress analysis of an upright cylindrical steel tank subjected to lateral excitation, tank is excited at and below resonance for peak ground acceleration (PGA) of 0.3g.

Problem Formulation

The cylindrical steel container of 90 m^3 capacity is considered, having a height to diameter ratio of 1.2 i.e., 5.5 m in height and 4.6 m in diameter with a tank shell thickness of 5mm.
Harmonic base excitation is applied at tank base represented mathematically as X(t)= DSin ω t where D is the horizontal magnitude of dynamic loading and ω is the excitation frequency. Fluid is filled up to 80 percent by volume. This dynamic loading is equivalent to seismic excitation of 0.3g whereas the two excitation frequencies are applied i.e., below resonance and at convective resonance. The tank foundation is supported from the ground. In this article full-scale dynamic analysis is performed instead of scaled analysis performed by Goudarzi and his colleagues [11] and further by Mahmood Hosseini for dynamic analysis of rectangular tanks [12].

Theoretical Evaluation Procedure

When the upright cylindrical liquid storage container is subjected to harmonic excitation, the fluid inside it will displace and its free surface also oscillates which is represented in vector form by Euler's equation of motion of the fluid. Euler's equation is derived from Newton's second law of motion i.e. $F = ma$, Where *F* is a vector quantity and it may comprise of gravity forces, pressure forces, viscous forces, turbulent forces and compressibility forces. When viscous forces, turbulent forces and compressibility forces become zero the equation of motion is called Euler's equation, therefore Euler's equation includes only forces due to gravity and pressure. The Euler's equation of motion of the fluid is written in vector form as

$$-\frac{1}{\rho}\nabla P - \nabla(gy) = \frac{\partial}{\partial t}q + (q.\nabla)q \qquad (1)$$

Where *q* is the fluid velocity, $\partial q/\partial t$ is the local acceleration, $(q.\nabla)q$ is the convective acceleration, *P* is the fluid pressure, ρ is the fluid density and *gy* is the gravitational potential.

By putting $(q.\nabla)q = \frac{1}{2}\nabla q^2$ and $q = \nabla\phi$ into equation (1) we get

$$\nabla(\frac{P}{\rho} + \frac{1}{2}q^2 + gy - \frac{\partial\phi}{\partial t}) = 0 \qquad (2)$$

Integration of equation (2) gives

$$\frac{P}{\rho} + \frac{1}{2}q^2 + gy - \frac{\partial\phi}{\partial t} = c(t) \qquad (3)$$

This equation is called Kelvin's equation for an unsteady fluid flow and ϕ is the potential function of time and space. By introducing the continuity condition $(\nabla.q) = 0$ and $(q.\nabla)q = \frac{1}{2}\nabla q^2$ into equation (3) yields Laplace's equation represented by equation (4a).

$$\nabla^2 \phi = 0 \tag{4a}$$

$$\frac{\partial \phi}{\partial r}|_{r=R} = 0, \frac{\partial \phi}{\partial y}|_{y=-h} = 0 \tag{4b, c}$$

$$g\eta - \frac{\partial \phi}{\partial t} + \ddot{X} r \cos\theta = 0, \ \frac{\partial \eta}{\partial t} + \frac{\partial \phi}{\partial z} = 0 \quad at \quad z = \eta(r,\theta,t) \tag{4d}$$

The dynamic and kinematic free surface conditions can be written as equation (4d) [13].

$$g\frac{\partial \phi}{\partial z} + \frac{\partial^2 \phi}{\partial t^2} = \dddot{X} r \cos\theta \tag{5}$$

By solving and applying boundary conditions represented in equation (4b, c) to equation (4a) gives the solution of the potential function ϕ represented by equation (6) [13].

$$\phi = \sum_{n=1}^{\infty} [A_{1n}(t)\cos\theta + B_{1n}(t)\sin\theta] J_1(k_n r) \frac{\cosh[k_{1n}(z+h)]}{\cosh k_{1n} h} \tag{6}$$

Where A_{1n} and B_{1n} are time-dependent functions and evaluated by satisfying equation (5) and substituted in equation (6) resulted in tank total potential function is shown below.

$$\phi = -D\omega\cos\theta\cos\omega t \times \left\{ r + \sum_{n=1}^{\infty} \frac{2R}{\left(\lambda_{1n}{}^2 - 1\right)} \frac{\omega^2}{\left(\omega_{1n}{}^2 - \omega\right)} \frac{J_1\left(\lambda_{1n} r / R\right)}{J_1\left(\lambda_{1n}\right)} \frac{\cosh\left[\lambda_{1n}\left(y+h\right)/R\right]}{\cosh \lambda_{1n} h / R} \right\} \tag{7}$$

The hydrodynamic pressure at any point below resonance i.e., excitation frequency below sloshing frequency, due to sloshing of liquid may be determined from the following pressure equation.

$$P = \rho \frac{\partial \phi}{\partial t} \tag{8}$$

By putting the value of potential function represented by equation (7) in equation (8) we get

$$P = \rho D\omega^2 \cos\theta\sin\omega t \times \left\{ r + \sum_{n=1}^{\infty} \frac{2R}{\left(\lambda_{1n}{}^2 - 1\right)} \frac{\omega^2}{\left(\omega_{1n}{}^2 - \omega\right)} \frac{J_1\left(\lambda_{1n} r / R\right)}{J_1\left(\lambda_{1n}\right)} \frac{\cosh\left[\lambda_{1n}\left(y+h\right)/R\right]}{\cosh \lambda_{1n} h / R} \right\} \tag{9}$$

Therefore, the value of hydrodynamic pressure and then total sloshing pressure which is the summation of hydrodynamic pressure and hydrostatic pressure $\rho g y$ may be calculated from equation (9) [13]. Similarly, the equivalent expressions for calculation of impulsive and convective pressures are represented in Eurocode-8 [6] i.e.

$$P_{\mathrm{Im}} = \left[2\sum_{n=0}^{\infty} \frac{(-1)^n}{I_1^{'}(v_n / \gamma) v_n^2} \cos(v_n \frac{y}{h}) I_1(\frac{v_n}{\gamma}\frac{r}{R}) \right] \rho h \cos\theta A_g(t) \tag{10}$$

Where $v_n = \frac{2n+1}{2}\pi$, $\gamma = h/R$, $I_1(.)$ and $I_1^{'}(.)$ are first-order Bessel function (modified).

$$P_c = \rho \sum_{n=1}^{\infty} \psi_n \cosh(\lambda_n \gamma \frac{y}{h}) J_1(\lambda_n \frac{r}{R}) \cos\theta A_{cn}(t) \tag{11}$$

Where $\psi_n = \dfrac{2R}{\left(\lambda_n^2 - 1\right) J_1(\lambda_n)\cosh(\lambda_n \gamma)}$ J_1 is the Bessel function of the first order, λ_1 equals 1.8 and in reaction to ground motion, the fluid movement is represented by convective acceleration term $A_{cn}(t)$. The value of sloshing pressure will be calculated from equation (9) e.g., for the calculation of pressure occurs at tank wall put r =R (Tank Inner Radius), θ= 0 or 180°, and y= value of interest along with the height of tank wall. According to Eurocode-8, the same value has been calculated by transforming tank base displacement excitations to ground acceleration time history, $A_g(t)$. These Equations i.e., equation (9) derived from classical potential flow approach [13] and equation (10) & (11) based on Eurocode-8 [6] provides the basis of validating the FEM modelling procedure for the same case and permits its subsequent application for resonance case for flat and conical roof cylindrical tank.

FE Modelling Procedure

The dynamic analysis is performed using Finite Element Modelling software ANSYS which starts with the evaluation of convective frequency and its respective mode shape after that detailed FSI analysis is performed that is done using ANSYS Workbench package for evaluation of structural response due to lateral seismic loadings.

Modal Analysis

In Modal analysis, the fluid is modelled using eight nodded brick type fluid elements with three DOF at each node named FLUID80. The sloshing frequency is calculated theoretically using the expression given below.

$$\omega_{cn} = \frac{1}{2\pi}\sqrt{g\frac{\lambda_n}{R}\tanh(\lambda_n \gamma)} \tag{12}$$

Where ω_{cn}, λ_n, R, g & γ represents the convective frequency, Bessel function solution, tank radius, gravitational acceleration, and slenderness ratio. 1st convective frequency is evaluated using tank design code frequency expression represented by equation (12) [6] and is computed as 2.806 rad/sec whereas numerically estimated as 2.808 rad/sec (see **Figure 1**). The first three convective frequencies are listed below in **Table 1** (Convective Frequencies).

Table 1 (Convective Frequencies)

Mode no	1	2	3
Convective Frequency (rad/sec)	2.808	3.619	4.055

The first convective frequency is used to excite the lateral harmonic load.

Fluid-Structure Interaction Analysis

In FSI analysis sloshing assessment is performed in the CFX module and ANSYS Transient is used to study the seismic response of the tank. The outcomes of CFX i.e., hydrodynamic pressures resulting due to sloshing are used as input load to the tank inner structure along with seismic excitations.

Fluid Analysis

The fluid is modelled using solid elements having uniform structured hexahedral meshing (see **Figure 2**) and fluid is incompressible and viscosity effects are negligible. The density of water is 1000 kg/m^3. Modelling of sloshing is done using the Volume of the fluid model (VOF) along with the k-epsilon turbulence model to capture sloshing turbulence.

Mesh and time step size independence study has been performed and, on this basis, the selected grid and timestep size indicates very small variations in the CFX results i.e., less than 5%.

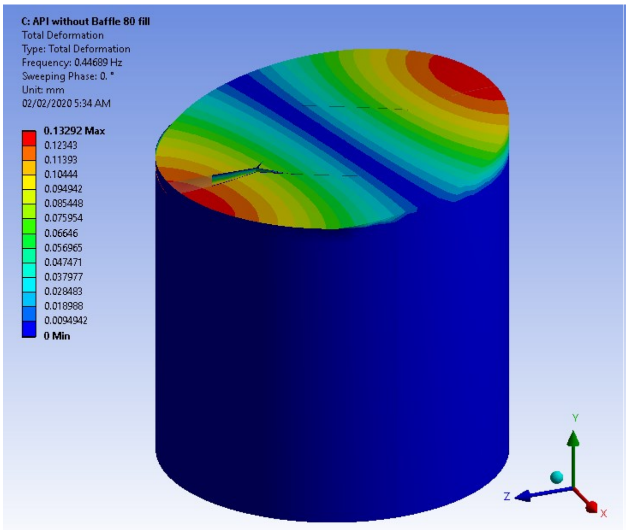


Figure 1 First Sloshing Frequency

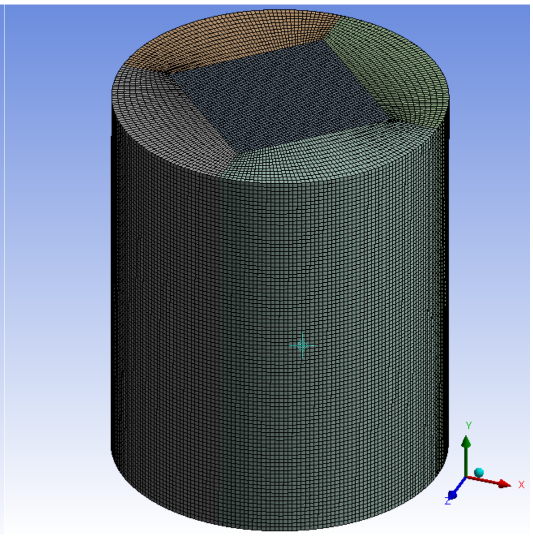


Figure 2 Fluid Element Meshing

Fluid Analysis below Resonance

Below resonance analysis is performed by exciting the tank base with the frequency of 0.5 ω_{cn}. The water contained in the tank moves in response to loading and exerts pressure on the wall of the tank. The time history of bottom, impulsive and convective pressure are evaluated from the fluid analysis. IITK-GSDMA Guidelines for seismic design of liquid storage tanks [4] is used for the calculation of impulsive and convective heights. These pressure variations have also been evaluated from equations (10) & (11) which are valid for below resonance loading for flat roof cylindrical tanks.

Verification of Numerical Model

Numerical verification is essentially required so that the numerical models can predict the responses with reasonable precision and accuracy. For this reason, the pressure variations at convective, impulsive height and tank wall bottom estimated from the fluid analysis were compared with the Eurocode 8. The comparison shows very good agreement of results having an average percentage error of only 1.78%. The same validation procedure under harmonic and seismic excitations have been used in the author's previous research articles [14,15]
These results have been shown graphically below in **Figure 3**. This validates our CFD model as there is no considerable difference in results.

Fluid Analysis at Convective Resonance

The resonant excitation is applied to the tank base for 4 seconds as water waves fully rise to the tank roof on either side of tank walls within this duration. It is observed that the water free surface moves violently and smashes against the tank roof and walls. As the water excites at resonance and it has a maximum slosh wave height of about 5.48 m at 2.39 seconds and exerts hydrodynamic pressure on the inner side of the tank roof.

Therefore when the tank is subjected to horizontal harmonic loading excited at a first convective frequency the wave moves up and down in orthogonal direction i.e. in our case free surface oscillates along y-direction in to and fro fashion, as time goes on, the amplitude of wave increases, when it increases the sloshing pressure decreases whereas when it returns to mean position sloshing pressure goes up to some maximum value and this is the moment when the tank is prone to failure during an earthquake event.

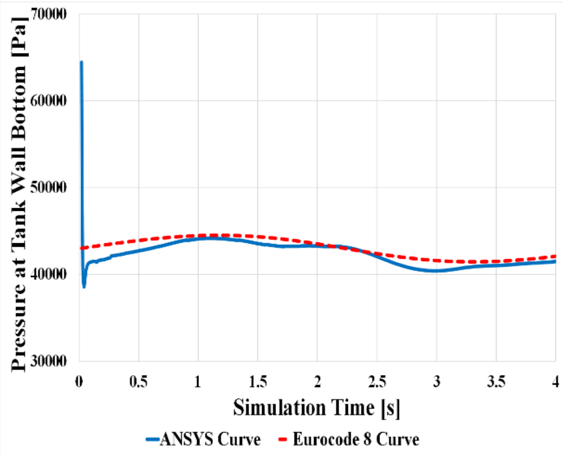


Figure 3 Sloshing Pressure Vs Simulation

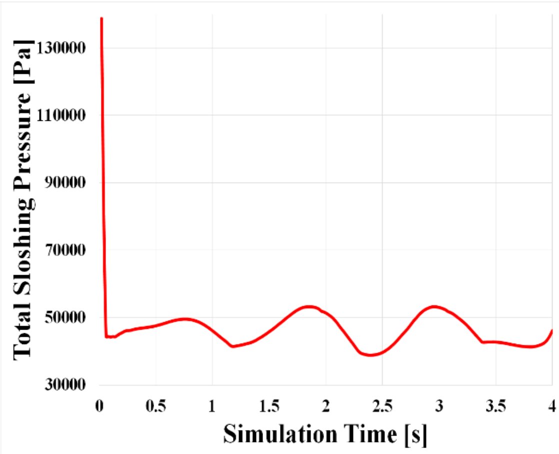


Figure 4 Sloshing Pressure Vs Analysis

At maximum sloshing wave height, the hydrodynamic pressure has the lowest value amongst the values of hydrodynamic pressure-time history. These pressure peaks have been shown above in **Figure 4**. The water in the tank consists of two masses one is called convective and the other one is impulsive mass. When the water free surface rises, its amplitude increases, results in low hydrodynamic pressure, **Figure 5** shows the maximum amplitude of the sloshing wave at which the hydrodynamic pressure is at its lowest value whereas when the water free surface moves back to its mean position the value of hydrodynamic pressure rises as all of the convective mass adds up to impulsive mass and this pulls the whole mass of fluid against the wall of the tank resulting in maximum pressure at that instant. **Figure 6** clearly shows the position of water free surface when sloshing pressure is at its peak value called Maximum Sloshing Pressure (MSP).

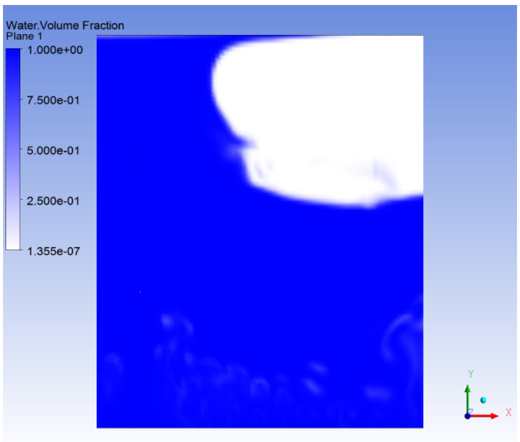


Figure 5 Maximum Sloshing Wave Height

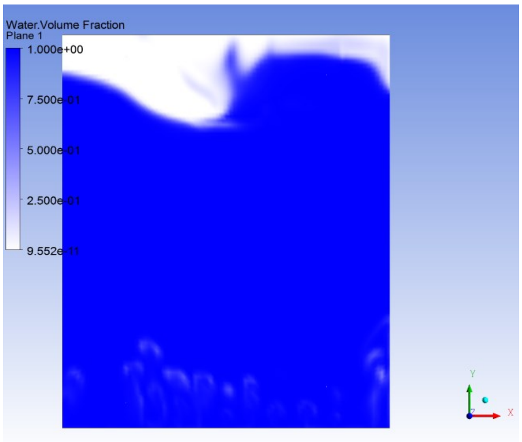
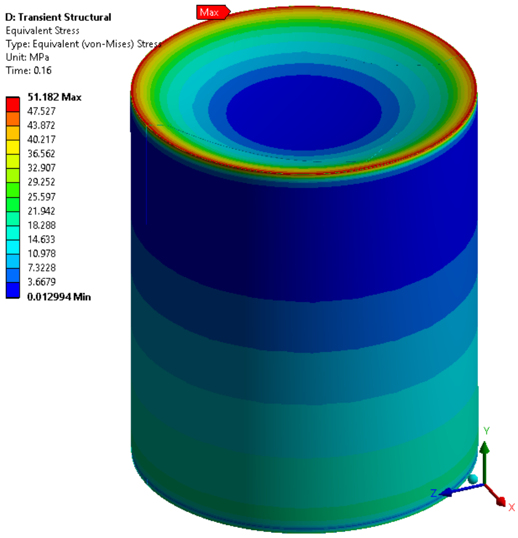


Figure 6 Wave Position at MSP

Structural Analysis

In structural analysis, the storage tank is made up of structural steel having the following material properties i.e., Modulus of elasticity of 2.1E+11, Poisson ratio of 0.3 and density of 7850 Kg/m^3 [16]. The steel tank is modelled as a shell element. Fixed support is applied to the base of the tank and CFX total pressure-time history is used for structural analysis of flat roof cylindrical tank. For 50% of the convective frequency, the maximum stress produced is about 51 MPa at the top circular edge of the tank flat roof. The tank structure vibrates in response to sloshing resulting in abrupt stress variations, but the tank remains safe up to the analysis time of 4 seconds. But when the tank is operated at first convective or the sloshing frequency it crosses yield strength of tank material crosses in just 2.04 sec and reaches the maximum stress value of 281.5 MPa. The maximum stress in a flat roof tank at and below resonance is shown in **Figures 7 & 8**.



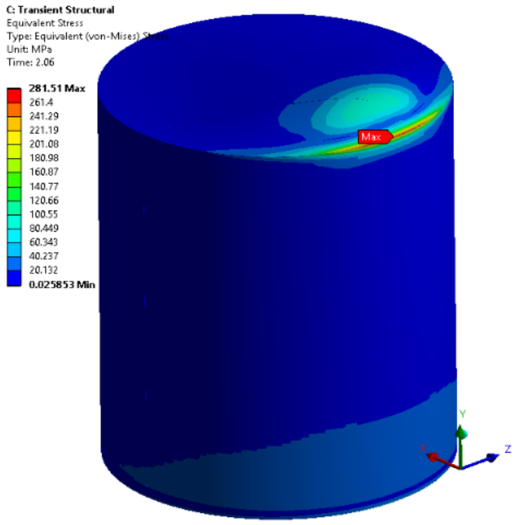


Figure 7 Stress Below Resonance Figure 8 Tank yielding at Resonance

This indicates that at lateral resonance loading of 0.3g the flat roof cylindrical tank fails to withstand seismic activity up to about 2 seconds. The location of maximum stress at below resonance is at the top circular edge of the flat roof and at resonance, the location is at the roof edge. This indicates that when the flat closed cylindrical tank is operated at first convective resonance, it crosses material yield stress (250 MPa) in just 2 sec. The Von-Mises Stresses in flat roof cylindrical tank at and below resonance has been shown graphically in **Figure 9**.

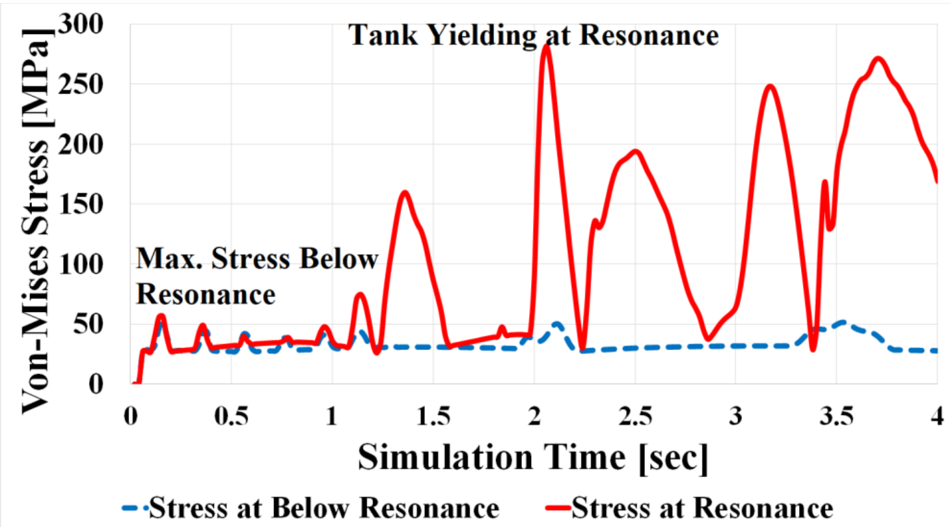


Figure 9 Von-Mises Stress in Flat Roof Tank at and Below Resonance.

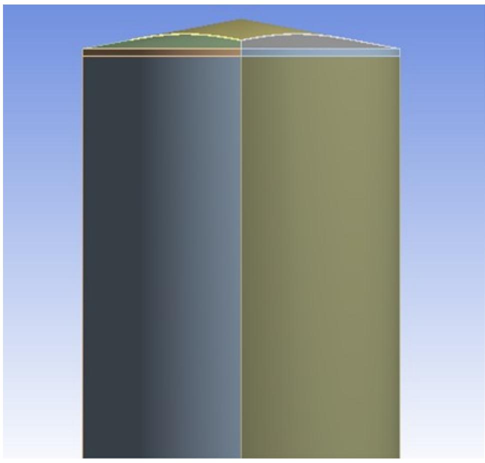
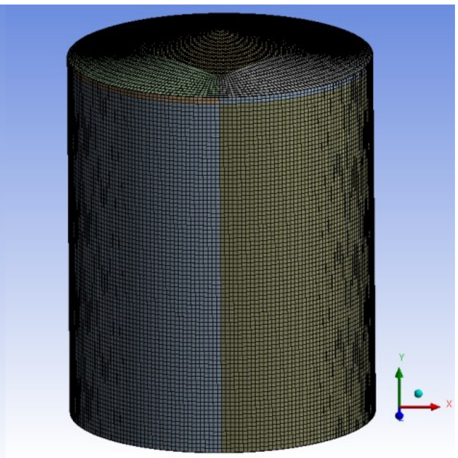



Figure 10 Conical Roof Cylindrical Tank Figure 11 Conical Roof Tank Meshing

Geometrical Modifications

For the tank under resonance consideration, the location of maximum stress is at the roof edge due to large stress concentrations near the top edge. As a flat roof design produces excessive stresses, therefore roof design is improved from flat to conical (see **Figures 10 & 11**) and the same analysis settings have been applied.

For conical roof design having roof angle of 30°, it is observed that Equivalent Stress has the maximum value of 108.6 MPa at 3.19 sec instead of crossing yield strength of 250 MPa as in case of flat roof tank within 4 sec. The comparison of Von-Mises stress of flat roof and conical roof design at resonance has been shown graphically in **Figure 12.**

It is observed that the conical roof tank withstands the seismic activity of 0.3g even at resonance for 4 seconds and remains safe. Therefore, the tank structural strength increases up to 2.6 times as compared to the maximum stress that occurred in a flat roof tank at resonance. The location of Maximum Stress is the same as in a flat roof tank i.e., at the top edge of the tank roof.

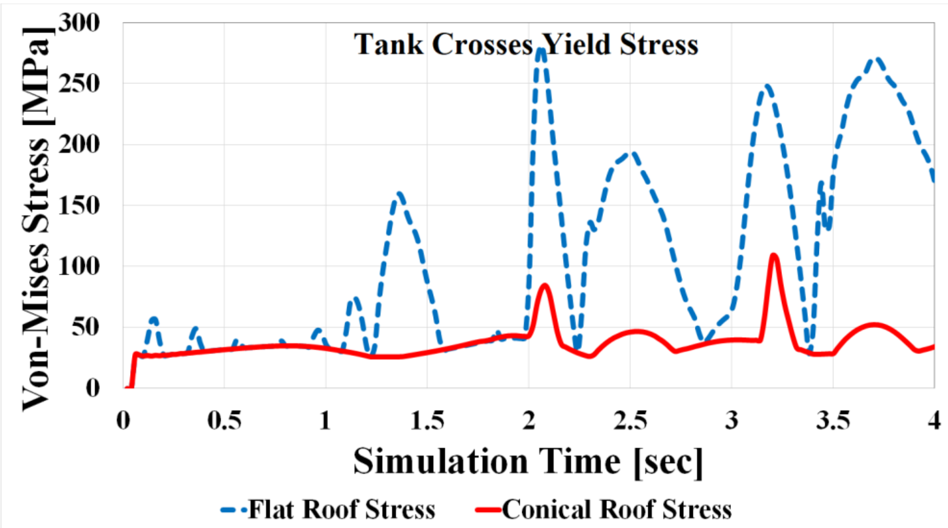


Figure 12 Comparison of Stress in Flat & Conical Roof Tank at Resonance.

Figure 13 shows that at the duration at which the flat roof tank fails, the conical tank is in safe stress limits i.e., 81 MPa at 2.055 sec and maximum stress is also below the yield strength of the tank as shown in **Figure 14**. Therefore, a conical roof tank provides a much safer design to cope even with the first resonance loading and at a high value of seismic accelerations.

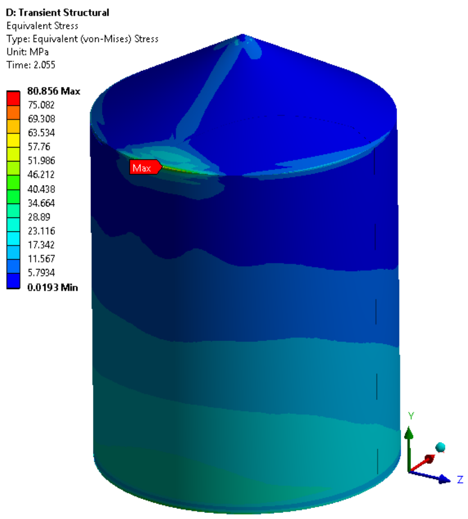


Figure 13 Stress in Conical Roof Tank

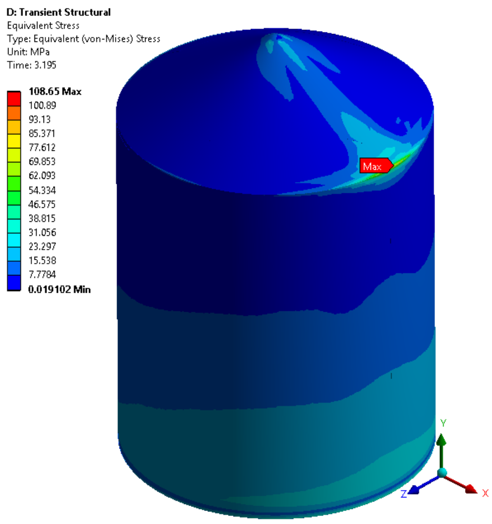


Figure 14 Maximum Stress in Conical Roof Tank

Conclusion

This paper presents the stress analysis of a cylindrical steel tank of 90 m^3 capacity subjected to seismic loading equivalent to Peak Ground Acceleration (PGA) of 0.3g excited at resonance. The stress levels at 50% of first convective frequency and at convective resonance for flat roof cylindrical tank have been evaluated and analysed and it is observed that at resonance tank with flat roof design crosses yield strength at resonance therefore the main aim of this research is to reduce the roof stresses by modifying the roof design from flat to the conical roof of 30^o roof angle. The maximum value of stress at resonance reaches about 281.5 MPa within almost 2 seconds of application of sinusoidal load and resulting in tank structural failure whereas for conical roof tank the maximum stress reaches about 108 MPa for the analysis of 4 sec. Therefore, conical roof design provides much better seismic resistance even at convective resonance, the maximum stress due to sloshing reduces about 2.6 times.

Nomenclature

P_{im}	Impulsive pressure
P_c	Convective pressure
MSP	Maximum Sloshing Pressure
J_1	Bessel function of the first order
PGA	Peak Ground Acceleration
$A_g(t)$	Ground acceleration time history
$A_{cn}(t)$	Convective acceleration time history
R	Tank radius
h	Fluid height
ω_{cn}	Convective Frequency
ρ	Fluid density
$\eta(r,\theta,t)$	Free surface Co-ordinates
γ	Slenderness ratio (h/R)
λ_n	Bessel function Solution
ω	Excitation frequency
P	Fluid pressure
q	Fluid Velocity

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